A Time-domain Discontinuous Galerkin Method for Solving Maxwell Equations with Kerr Nonlinearity

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Abstract— Kerr effects are observed when the response of a material to electromagnetic excitation nonlinearly depends on the magnitude of the electric field intensity. This property leads to several optical phenomena, such as self-phase modulation, self-focusing, and bistablity, with interesting engineering applications in optical communications and data processing/storage. Design of electromagnetic devices in these applications calls for simulation tools capable of modeling electromagnetic field interactions on arbitrarily-shaped geometries with nonlinear material properties.

Initial numerical methods developed to analyze Kerr effects have mainly focused on the finite difference time-domain (FDTD) scheme due to its straightforward implementation and low computational cost [1]. However, FDTD suffers from "staircasing" errors when it is applied to geometries with curved surfaces. As an alternative to FDTD, finite volume time-domain (FVTD) method has been put forward to solve Maxwell equations in Kerr nonlinear media [2]. Even though FVTD can account for curved surfaces more accurately, it still suffers low-order accuracy of the solution. Finite element time-domain (FETD) method has also been extended to account for Kerr nonlinearity since it can use high-order basis functions [3]. However, FETD requires inversion of a global sparse matrix at every time step which increases the overall computational cost and restricts the method's application to electrically large problems.

In this work, a time-domain discontinuous Galerkin (DGTD) method is formulated and implemented to simulate transient electromagnetic field interactions in Kerr nonlinear media. By combining the advantages of FVTD and FETD, DGTD discretizes curved surfaces accurately, provides high-order solution accuracy, easily incorporates h- and/or p-refinement strategies, and is easy to parallelize. To accurately account for the Kerr nonlinear Maxwell equations, a new numerical flux is derived from the Rankine-Hugoniot jump condition based on the approximate Riemann solver. Additionally, to suppress the nonphysical oscillations due to shock-wave generation in Kerr nonlinear medium, artificial viscosity together with a trouble-cell indicator is introduced within the proposed scheme. Numerical examples, which demonstrate the applicability and the accuracy of the proposed method, will be presented during the talk at the conference.

REFERENCES

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